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EFFECTIVE WEAK NON-LEPTONIC HAMILTONIANS FOR s, c, b & t DECAY

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ABSTRACT

This letter presents the results of calculations of effective weak non-leptonic Hamiltonians for s, c, b and t decay. It covers all aspects of the flavour changing sector of weak non-leptonics mediated by W exchange in the standard model with n quark generations.

*In the original preprint the figure captions of tables 2 & 3 were inadvertently transposed. We have taken this opportunity to correct a number of other minor errors as well as rewording certain passages to improve their clarity.

We have calculated the effective Hamiltonians for weak non leptonic falvour changing decays of s, c, b and t quarks to first order in G_W, and one loop order in QCD. In this letter we summarise our methods and present our results. Full details will be published elswhere (Miller and McKellar, 1981a).

The construction of the effective Hamiltonian for weak non leptonic processes is an application of the Applequist-Carazzone (1975) decoupling theorem to the non Abelian Gauge Field theory (NAGFT) $SU(3)_c \times SU(2) \times XU(1)$, involving explicit decoupling of the W boson (Gaillard and Lee 1974, and Altarelli and Maiami 1974) and the heavy quarks (Gilman and Wise 1979, Wise 1980). Since this work there has been considerable progress in understanding the decoupling theorem and effective Lagrangians of NAGFTs (see for example Ovrut and Schnitzer 1981a, b, Kazama and Yao 1980, 1981, Weisberger 1981). Much of this work was developed with an eye to applications to Grand Unified Theories. We have developed results for the case of broken flavour symmetries which are appropriate to the present application (Miller and McKellar 1981b, c).

We find that the order in which the heavy particles are decoupled influences the final results. We therefore decouple particles sequentially in order of decreasing mass. The position of the W in the quark mass spectrum is thus significant. We assume that there are six quarks lighter than the W, and that any other quarks are heavier. In fact it turns out not to matter too much if the b' is lighter or heavier than the W.

The flavour changing processes may be divided into two classes which we call penguin free and penguin generating, depending on whether penguin operators contribute to the process or not. Penguin operators occur when the fundamental four fermion interaction can be written (by Fierz Transformation if necessary) as a product of two fermion operators in charge conserving form in which only one of the operators involves a change of flavour. Thus penguin generating flavour changing processes are 'Cabibbo forbidden' in the general sense.

Our work follows the general procedures laid down by Gilman and Wise (1979) in their discussion of $\Delta S=1$ decays, but we have introduced a number of refinements. We treat the general flavour changing process in a matrix formulation which makes it possible to discuss s decays (Gilman and Wise 1979, Wise 1980, Guberina and Peccei 1980), c decays (Hayashi et al, 1980) b decays (Ponce 1981) and t decays (which seem to have escaped attention heretofore) in a unified way which is conceptually and computationally convenient. Consistancy in performing Laurent expansions to a uniform order requires us to employ a second order parameterisation of the running coupling constant and a first order parameterisation of running quark mass. These in turn demand inclusion of scheme dependence on R.G. invariants. Furthermore we include a full effective theory dependence in the R.G. invariants, this has not been done before. We have also taken particular care to ensure at all times well defined running parameters at relevant mass scales.

Throughout the work we have employed dimensional regularisation, the minimal subtraction renormalisation scheme, and we have worked in the Landau ($\alpha = 0$) gauge.

We consider n generations of quarks in the standard $SU(3)_c \times SU(2) \times U(1)$ model. Instead of SU(2) spinors $\begin{pmatrix} u \\ d \end{pmatrix}$, $\begin{pmatrix} c \\ s \end{pmatrix}$, ..., $\begin{pmatrix} q_{n+} \\ q_{n-} \end{pmatrix}$ we introduce upper and lower "horizontal" vectors $\underset{\sim}{q}_+^T = (u, c, \ldots, q_{n+})$ and $\underset{\sim}{q}_-^T = (d, s, \ldots, q_{n-})$ to describe the quarks.

The generalised Kobayashi-Maskawa (1973) matrix V transforms $\underset{\sim}{q}_-$ into $\underset{\sim}{q}'_- = V\underset{\sim}{q}_-$, which enters the weak current J_μ^+ through $J_\mu^+ = (\bar{q}'^i_- q^i)_L$ where we adopt the notation $(\bar{q}^i q^i)_{L(R)} = (\bar{q}^i \gamma^\mu a(a') q^i)$ with $a[a'] = \frac{1}{2}(1-[+]\gamma_5)$. Summation on the colour index i is implied. The basic W-quark interaction Hamiltonian is

$$\mathcal{H}_I = \frac{g_W}{\sqrt{2}} J_\mu^+ W_-^\mu + \text{h.c.} \tag{1}$$

Following the decoupling of the W and all quarks heavier than t, the effective weak non leptonic flavour changing Hamiltonian naturally decomposes in the fashion

$$\mathcal{H}^{\Delta}_{eff} = \mathcal{H}^{\Delta}_{eff}\,(P.F.) + \mathcal{H}^{\Delta}_{eff}\,(P.G.)$$

The penguin free and penguin generating cases are somewhat different and we treat each in turn. Specialising to flavour changing decays of quarks s,c,b & t, we find that the effective Hamiltonian for penguin free flavour decays, $\mathcal{H}^{\Delta}_{eff}$ (P.F.), can be written after decoupling as

$$\mathcal{H}^{\Delta}_{eff}\,(P.F.) = \frac{4G_W}{\sqrt{2}} \sum_{\theta,\phi=1}^{3}{}' \sum_{\psi,\chi=1}^{3}{}' \left[\underset{\sim}{A}^{\theta\phi\psi\chi}\,(1,0) \right]^{T} \exp\left[S_6(M_W,\mu)\,\underset{=}{\tilde{\gamma}}_o^{(6)} \right] \times {}_6\underset{\sim}{\mathcal{O}}^{\theta\phi\psi\chi} \qquad (2)$$

The prime on the summation signs indicates that the case where the indices being summed are equal is to be omitted. The 4 fermion operators ${}_k\underset{\sim}{\mathcal{O}}^{\theta\phi\psi\chi}$ are defined in a k flavour effective theory as

$${}_k\underset{\sim}{\mathcal{O}}^{\theta\phi\psi\chi} = \begin{pmatrix} \left(\bar{q}^i_{\theta+}\; q^i_{\phi+}\right)_L \left(\bar{q}^j_{\psi-}\; q^j_{\chi-}\right)_L \\ \left(\bar{q}^i_{\theta+}\; q^j_{\phi+}\right)_L \left(\bar{q}^j_{\psi-}\; q^i_{\chi-}\right)_L \end{pmatrix} \qquad (3)$$

We do not need to eliminate heavy quarks lighter than the W since that part of (2) describing a particular flavour decay has no explicit reference to such heavier flavours. Note that the effects of a quark heavier than the W enter through their influence on $\underset{=}{V}$ and through their influence on the parameters of the effective lagrangian of the 6 quark theory. This influence is taken into account in the empirical values used for the parameters. The function S_k (P,Q) is defined in the k flavour effective theory with running coupling constant $\bar{g}_k(Q)$ (defined in the standard way). It is

$$S_k(P,Q) = \frac{1}{11 - 2k/3} \; \ell n \left(\frac{\bar{g}_k^2(P)}{\bar{g}_k^2(Q)} \right) \tag{4}$$

The matrix $\underset{=}{\tilde{\gamma}}_o^{(k)}$ is defined as the coefficient of $g_k^2/8\pi^2$ in the perturbation expansion of the anamolous dimension matrix for $\underset{k\sim}{\mathcal{O}}^{\theta\phi\psi\chi}$, which is a closed renormalisation set of operators at the one loop level.

Explicitly

$$\underset{=}{\tilde{\gamma}}_o^{(k)} = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} \tag{5}$$

The coefficient vector $[\, \underset{\sim}{A}^{\theta\phi\psi\chi}(1,0) \,]^T$ are the free field coefficients of the operators $\underset{k\sim}{\mathcal{O}}^{\theta\phi\psi\chi}$. They arise in the scaling limit through the use of the leading log approximation, and are given in terms of the generalised KM matrix elements $V_{\theta\psi}$ by

$$[\, \underset{\sim}{A}^{\theta\phi\psi\chi}(1,0) \,]^T = \left(0 \, , \, [V^\dagger]_{\psi\phi} \, V_{\theta\chi} \right) \tag{6}$$

The penguin generating case is a little more involved.

In discussing $\mathcal{H}^{\Delta}_{eff}$ (PG) we discuss each flavour separately writing

$$\mathcal{H}^{\Delta}_{eff}\,(PG) = \sum_{\phi=2}^{3} \left[\mathcal{H}^{\Delta}_{eff}\,(\phi-,PG) + \mathcal{H}^{\Delta}_{eff}\,(\phi+,PG) \right] + H.C.$$

After decoupling quarks heavier than the W (which as before do not appear explicitly in the result, although they influence $\underset{=}{V}$ and the parameters of the effective theory), and the W itself we decouple k additional quarks to obtain

$$\mathcal{H}^{\Delta}_{eff}(\phi\pm\ ;\ PG) = \frac{4G_W}{\sqrt{2}} \sum_{\theta=1}^{\phi-1} \sum_{\psi=1}^{3} \left[\underset{\sim}{A}^{\theta\psi}_{\phi\pm}(1,0) \right]^T$$

$$\times \exp\left\{ (S_6(m_W,\mu) - \tilde{S}_6)\ \underline{\underline{\tilde{Y}}}^{(6)}_o \right\}$$

$$\times \prod_{\ell=1}^{k-1} \left[\underline{\underline{B}}^{\theta\psi}_{6-\ell,\phi\pm}\ (1,0)\ \exp\left\{\left(\tilde{S}_{6-\ell+1} - \tilde{S}_{6-\ell}\right) \underline{\underline{\tilde{Y}}}^{(6-\ell)}_o \right\} \right]$$

$$\times \underline{\underline{B}}^{\theta\psi}_{6-k,\phi\pm}\ (1,0)\ \exp\left\{\tilde{S}_{6-k+1}\ \underline{\underline{\tilde{Y}}}^{(6-k)}_o\right\}_{6-k} \underset{\sim}{\mathcal{O}}^{\theta\psi}_{\phi\pm} \qquad (7)$$

The vector $\underset{k\sim\phi\pm}{\mathcal{O}}^{\theta\psi}$ contains a set of 4 fermion operators closed under renormalisation. It is in this set that penguin operators involving left handed and right handed currents appear.

We have explicitly

$$\underset{k\sim\phi\pm}{\mathcal{O}}^{\theta\psi} = \begin{bmatrix} \left(\bar{q}^i_{\theta\pm}\ q^i_{\phi\pm}\right)_L \left(\bar{q}^j_{\psi\mp}\ q^j_{\psi\mp}\right)_L \\ \left(\bar{q}^i_{\theta\pm}\ q^j_{\phi\pm}\right)_L \left(\bar{q}^j_{\psi\mp}\ q^i_{\psi\mp}\right)_L \\ \sum_{\ell=1}^{k} \left(\bar{q}^i_{\theta\pm}\ q^i_{\phi\pm}\right)_L \left(\bar{q}^j_{\ell}\ q^j_{\ell}\right)_L \\ \sum_{\ell=1}^{k} \left(\bar{q}^i_{\theta\pm}\ q^j_{\phi\pm}\right)_L \left(\bar{q}^j_{\ell}\ q^i_{\ell}\right)_L \\ \sum_{\ell=1}^{k} \left(\bar{q}^i_{\theta\pm}\ q^i_{\phi\pm}\right)_L \left(\bar{q}^j_{\ell}\ q^j_{\ell}\right)_R \\ \sum_{\ell=1}^{k} \left(\bar{q}^i_{\theta\pm}\ q^j_{\phi\pm}\right)_L \left(\bar{q}^j_{\ell}\ q^i_{\ell}\right)_R \end{bmatrix} \qquad (8)$$

Again i, j are colour indices, and the index ℓ labels quark components of the vector $\underset{\sim}{q}^T = (q_{1-}\ q_{1+}\ \cdots\cdots\ q_{n-}\ q_{n+})$. If $q_{\psi\pm}$ is a heavy quark which has been eliminated the first two elements of the vector $\underset{k\sim\phi\pm}{\mathcal{O}^{\theta\psi}}$ are undefined, but this is immaterial as these elements will always be multiplied by zero in eqn(7). It is precisely this artifice, enabling us to write $\underset{k\sim}{\mathcal{O}}$ as a 6 dimensional vector for all k which in turn allows us to write the result for $\hat{\mathcal{H}}_{eff}[\phi\pm,PG]$ in such a compact form, which moreover lends itself readily to computer evaluation.

$\tilde{\Upsilon}_o^{(k)}$ is defined as the coefficient of $g_k^2/8\pi^2$ in the perturbation expansion of the anomalous dimension matrix for $\underset{k\sim\phi\pm}{\mathcal{O}^{\theta\psi}}$ and is found to be,

$$\tilde{\Upsilon}_o^{(k)} = \begin{bmatrix} -1 & 3 & 0 & 0 & 0 & 0 \\ 3 & -1 & -1/9 & 1/3 & -1/9 & 1/3 \\ 0 & 0 & -11/9 & 11/3 & -2/9 & 2/3 \\ 0 & 0 & (3-k/9) & (-1+k/3) & (-k/9) & (k/3) \\ 0 & 0 & 0 & 0 & 1 & -3 \\ 0 & 0 & (-k/9) & (k/3) & (-k/9) & (-8+k/3) \end{bmatrix} \tag{9}$$

The free field coefficients A and $\underline{\underline{B}}$ are defined by

$$\left[A_{\phi-}^{\theta\psi}(1,0)\right]^T = \left(0,\ [V^\dagger]_{\theta\psi}\ V_{\psi\phi},0,0,0,0\right) \tag{10a}$$

$$\left[A_{\phi+}^{\theta\psi}(1,0)\right]^T = \left(0,\ V_{\theta\psi}\ [V^\dagger]_{\psi\phi},0,0,0,0\right) \tag{10b}$$

$$\left[\underline{\underline{B}}_{6-\ell,\phi-}^{\theta\psi}(1,0)\right]_{ij} = \delta_{ij}\left(1 - \delta_{\psi,\ 3-\frac{1}{2}(\ell-1)}[\delta_{i1} + \delta_{i2}]\right) \quad \text{for } \ell \text{ odd}$$

$$= \delta_{ij} \quad \text{for } \ell \text{ even} \tag{10c}$$

$$\left[\underset{=}{B}{}^{\theta\psi}_{6-\ell,\phi+}(1,0)\right]_{ij} = \delta_{ij} \qquad \text{for } \ell \text{ odd}$$

$$= \delta_{ij}\left\{1 - \delta_{\psi,3-\frac{1}{2}(\ell-2)}\left[\delta_{i1} + \delta_{i2}\right]\right\}\,,$$

$$\text{for } \ell \text{ even} \tag{10d}$$

The symbol $\tilde{S}_k$ is an abbreviation for

$\tilde{S}_k\left(m_k^{(k)},\mu\right)$, where the definition

$$\tilde{S}_k(P,Q) = \frac{1}{11 - 2k/3}\,\ell n\left(\frac{\tilde{g}_k^2(P)}{\tilde{g}_k^2(Q)}\right) \tag{11}$$

is in terms of a non-standard running coupling constant $\tilde{g}_k(Q)$ in the k flavour theory. The non-standard $\tilde{g}_k(Q)$, defined by

$$\ell n\left(\frac{Q}{\mu}\right) = \int_{g_k}^{\tilde{g}_k(Q)} \frac{1 - \gamma_m^{(k)}(x)}{\beta_k(x)}\,dx \tag{12}$$

(where $\gamma_m^{(k)}(x)$ is the anamolous mass dimension in the k flavour theory), is to be compared with the standard running coupling constant $\bar{g}_k(Q)$ defined by

$$\ell n\left(\frac{Q}{\mu}\right) = \int_{g_k}^{\bar{g}_k(Q)} \frac{dx}{\beta_k(x)}\;. \tag{13}$$

$\tilde{g}_k(Q)$ can be related to $\bar{g}_k(Q)$ through the running mass parameter $\bar{m}_k^{(i)}(Q)$ for the ith flavour in the k flavour theory. Introducing the renormalisation group invariants Λ_k and $\tilde{\Lambda}_k^{(i)}$ by

$$\ell n\,(Q/\Lambda_k) = \int^{\bar{g}_k(Q)} \frac{dx}{\beta_k(x)}\;,\text{ and} \tag{14}$$

$$\ell n\,(\bar{m}_k^{(i)}(Q)/\tilde{\Lambda}_k^{(i)}) = \int^{\bar{g}_k(Q)} \frac{\gamma_m^{(k)}(x)}{\beta_k(x)}\,dx, \tag{15}$$

we note that

$$\ln\left(\frac{Q}{\Lambda_k}\frac{\tilde{\Lambda}_k^{(k)}}{m_k^{(k)}}\right) = \int^{\tilde{g}_k(Q)} \frac{1-\gamma_m^{(k)}(x)}{\beta_k(x)}\,dx \qquad (16)$$

It follows that $\bar{g}_k$ and $\tilde{g}_k$ are the same function evaluated at different points,

$$\tilde{g}_k\left(Q' = \frac{m_k^{(i)}}{\bar{m}_k^{(i)}(Q)}Q\right) = \bar{g}_k(Q)\ , \qquad (17a)$$

and that $\qquad \tilde{g}_k(\mu) = \bar{g}_k(\mu) = g_k \qquad (17b)$

In performing our calculations it is necessary to take account of the dependence of Λ_k and $\tilde{\Lambda}_k^{(i)}$ on the renormalisation scheme (Baće 1978), and the effective theory dependence of these invariants (Miller and McKellar 1981c). This arises since the integrands in (14) & (15) are Laurent expanded to the same order as those integrands which arise in the scaling coefficients, i.e. up to and including x^{-1} pole terms. This gives

$$\frac{16\pi^2}{\bar{g}_k^2(Q)} = \left(11 - \frac{2k}{3}\right)\ln\left(\frac{Q^2}{\Lambda_k^2}\right) + \frac{102 - 38k/3}{11 - 2k/3}\ln\left(\frac{1}{\bar{g}_k^2(Q)}\right) \qquad (18)$$

and

$$\bar{m}_k^{(i)}(Q) = \tilde{\Lambda}_k^{(i)}\left(\frac{1}{\bar{g}_k^2(Q)}\right)^{-\frac{4}{11-2k/3}} \qquad (19)$$

Using the value $\Lambda_4^2 = 0.023\ \text{GeV}^2$ obtained for the M.S. scheme by Caneschi (1981) from a reanalysis of the data of de Groot et al (1979a,b,c), we have calculated the necessary invariants (Miller and McKellar 1981c). They are reproduced in table 1. To ensure that the iterative solution of eqn.(18) is valid we must choose the subtraction point above $\sqrt{2}$ GeV (Miller and McKellar 1981c). Subtraction point choices for s,c,b and t decay are made in the conventional fashion and are given in table 1. We use $m_W = 80$ GeV (Tran 1981).

Now we can present our numerical results for the effective Hamiltonians. The penguin free Hamiltonians, for decay of t, b and c quarks, are given by

$$\mathcal{H}^{t}_{eff}[PF] = \frac{4G_W}{\sqrt{2}} \sum_{\theta=1}^{2} \sum_{\psi,\chi=1}^{3}{}' [V^{\dagger}]_{\psi 3} V_{\theta\chi} \left(b_6\, {}_6\mathcal{O}_1^{\theta 3\psi\chi} + a_6\, {}_6\mathcal{O}_2^{\theta 3\psi\chi} \right) \tag{20a}$$

$$\mathcal{H}^{b}_{eff}[PF] = \frac{4G_W}{\sqrt{2}} \sum_{\psi=1}^{2} \sum_{\theta,\phi=1}^{2}{}' [V^{\dagger}]_{\psi\phi} V_{\theta 3} \left(b_5\, {}_6\mathcal{O}_1^{\theta\phi\psi 3} + a_5\, {}_6\mathcal{O}_2^{\theta\phi\psi 3} \right) \tag{20b}$$

$$\mathcal{H}^{c}_{eff}[PF] = \frac{4G_W}{\sqrt{2}} \sum_{\psi,\chi=1}^{2}{}' [V^{\dagger}]_{\psi 2} V_{1\chi} \left(b_4\, {}_6\mathcal{O}_1^{12\psi\chi} + a_4\, {}_6\mathcal{O}_2^{12\psi\chi} \right) \tag{20c}$$

The primed summation sign was defined following eqn.(2). There is no penguin free Hamiltonian for s decay. The coefficients a_k and b_k given in Table 1.

We can compare our Hamiltonian for c quark decay with the familiar result of Cabibbo and Maiani (1978) by noting that their $f_{\pm}$ parameters are related to our a_4 and b_4 by

$$f_{\pm} = a_4 \pm b_4 \tag{21}$$

The 6 quark results for $f_{\pm}$ given by Cabibbo and Maiani are

$$f_{+}^{(CM)} = 0.68 \quad \text{and} \quad f_{-}^{(CM)} = 2.15 \;, \tag{22}$$

whereas we obtain

$$f_{+} = 0.76 \quad \text{and} \quad f_{-} = 1.74 \;. \tag{23}$$

The relationship $f_+ = (f_-)^{-1/2}$ is still satisfied, but the QCD enhancement of the SU(4) 20 operator is reduced, reflecting the smaller value of Λ_k^2 in this calculation.

The effective Hamiltonians in the penguin generating sector are

$$\mathcal{H}^t_{eff}[PG] = \frac{4G_W}{\sqrt{2}} \sum_{\theta=1}^{2} \left\{ \sum_{i=1}^{2} \sum_{\psi=1}^{3} C^{\theta\psi}_{t,i} \left[{}_6\underset{\sim}{\mathcal{O}}^{\theta\psi}_{3+}\right]_i + \sum_{i=3}^{6} D^{\theta}_{t,i} \left[{}_6\underset{\sim}{\mathcal{O}}^{\theta 1}_{3+}\right]_i \right\} \tag{24a}$$

$$\mathcal{H}^b_{eff}[PG] = \frac{4G_W}{\sqrt{2}} \sum_{\theta=1}^{2} \left\{ \sum_{i=1}^{2} \sum_{\psi=1}^{2} C^{\theta\psi}_{b,i} \left[{}_5\underset{\sim}{\mathcal{O}}^{\theta\psi}_{3-}\right]_i + \sum_{i=3}^{6} D^{\theta}_{b,i} \left[{}_5\underset{\sim}{\mathcal{O}}^{\theta 1}_{3-}\right]_i \right\} \tag{24b}$$

$$\mathcal{H}^c_{eff}[PG] = \frac{4G_W}{\sqrt{2}} \left\{ \sum_{i=1}^{2} \sum_{\psi=1}^{2} C^{1\psi}_{c,i} \left[{}_4\underset{\sim}{\mathcal{O}}^{1\psi}_{2+}\right]_i + \sum_{i=3}^{6} D^{1}_{c,i} \left[{}_4\underset{\sim}{\mathcal{O}}^{11}_{2+}\right]_i \right\} \tag{24c}$$

$$\mathcal{H}^s_{eff}[k,PG] = \frac{4G_W}{\sqrt{2}} \left\{ \sum_{i=1}^{2} \sum_{\psi=1}^{k-2} C^{1\psi}_{s(k),i} \left[{}_k\underset{\sim}{\mathcal{O}}^{1\psi}_{2-}\right]_i + \sum_{i=3}^{6} D^{1}_{s(k),i} \left[{}_k\underset{\sim}{\mathcal{O}}^{11}_{2-}\right]_i \right\}$$

$$\text{for } k = 3 \text{ or } 4 \tag{24d}$$

Because the c quark is regarded as being on the borderline between light and heavy quarks we have allowed two possibilities in eqn.(24d). If k = 3 the c is treated as a heavy quark and eliminated. If k = 4 the c quark is retained as a light quark and the effective Hamiltonian for s quark decay is to be used in a 4 flavour theory.

Results for the coefficients C and D are given in tables 2 to 6, where reductions to linearly independent sets of operators have been performed.

In the case of $\mathcal{H}^{s}_{eff}$ (3,PG) we can compare our results with those of Gilman and Wise (1979). This we do in a separate publication (Miller and McKellar 1981d). We find the contribution of penguin diagrams to the CP violation parameter ε'/ε is significantly increased.

We conclude by remarking that, while full details of the calculation will be given elsewhere, this letter provides enough for the reader to make his own model by varying parameters. The next major step in the application of these Hamiltonians to weak processes is the difficult task of evaluation of matrix elements of four fermion operators. This we leave for future work.

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T A B L E S

TABLE 1. Renormalisation group invariants Λ_k^2 and $\tilde{\Lambda}_k^{(k)^2}$ calculated in the MS scheme with Λ_4^2 as input. $\tilde{\Lambda}_k^{(k)^2}$ are based on threshold quark masses $m^{(4)} = 1.6$ GeV, $m^{(5)} = 4\ 7$ GeV and $m^{(6)} = 35$ GeV. μ_k represent conventional subtraction point choices for s, c, b and t decay, respectively corresponding to k = 3, 4, 5, and 6. a_k and b_k are coefficients for the penguin free Hamiltonians in (20)

k	Λ_k^2 (GeV^2)	$\left[\tilde{\Lambda}_k^{(k)}\right]^2$ (GeV^2)	μ_k (GeV)	a_k	b_k
3	0.026	-	$\sqrt{2}$	-	-
4	0.023	0.862	1.6	1.250	-0.492
5	0.017	8.83	4.7	1.124	-0.279
6	0.0080	671	35.0	1.022	-0.061

TABLE 2. Coefficients $C^{1\psi}_{s(3),i}$ and $D^{1}_{s(3),i}$ of the operator expansion of eqn.(24d) for the penguin generating Hamiltonian for s decays. The c quark is regarded as heavy and is decoupled from the theory.

i	ψ	$C^{1\psi}_{s(3),i}$
1	1	$-0.4727[V^{+}]_{11} V_{12} + 0.0451[V^{+}]_{12} V_{22} + 0.0056[V^{+}]_{13} V_{32}$
2	1	$1.2186[V^{+}]_{11} V_{12} - 0.0451[V^{+}]_{12} V_{22} - 0.0056[V^{+}]_{13} V_{32}$

i	$D^{1}_{s(3),i}$
3	$-0.0255[V^{+}]_{11} V_{12} - 0.0204[V^{+}]_{12} V_{22} - 0.0017[V^{+}]_{13} V_{32}$
4	0
5	$0.0137[V^{+}]_{11} V_{12} + 0.0111[V^{+}]_{12} V_{22} + 0.0010[V^{+}]_{13} V_{32}$
6	$-0.0802[V^{+}]_{11} V_{12} - 0.0718[V^{+}]_{12} V_{22} - 0.0125[V^{+}]_{13} V_{32}$

TABLE 3. Coefficients $C^{1\psi}_{s(4),i}$ and $D^{1}_{s(4),i}$ of the operator expansion of eqn.(24d) for the penguin generating Hamiltonian for s decays. The c quark is regarded as light and is not decoupled from the theory.

i	ψ	$C^{1\psi}_{s(4),i}$
1	1	$-0.4735[V^+]_{11} V_{12} + 0.0523[V^+_{12}] V_{22} + 0.0055[V^+]_{13} V_{32}$
	2	$0.0523[V^+]_{11} V_{12} - 0.4735[V^+]_{12} V_{22} + 0.0055[V^+]_{13} V_{32}$
2	1	$1.2194[V^+]_{11} V_{12} - 0.0523[V^+]_{12} V_{22} - 0.0055[V^+]_{13} V_{32}$
	2	$-0.0523[V^+]_{11} V_{12} + 1.2194[V^+]_{12} V_{22} - 0.0055[V^+]_{13} V_{32}$

i	$D^{1}_{s(4),i}$
3	$-0.0250[V^+]_{11} V_{12} - 0.0250[V^+]_{12} V_{22} - 0.0016[V^+]_{13} V_{32}$
4	0
5	$0.0135[V^+]_{11} V_{12} + 0.0135[V^+]_{12} V_{22} + 0.0010[V^+]_{13} V_{32}$
6	$-0.0794[V^+]_{11} V_{12} - 0.0794[V^+]_{12} V_{22} - 0.0124[V^+]_{12} V_{32}$

TABLE 4. Coefficients $C^{1\psi}_{c,i}$ and $D^{1}_{c,i}$ of the operator expansion of the penguin generating Hamiltonian for c decays of eqn.(24c).

i	ψ	$C^{1\psi}_{c,i}$
1	1	$-0.4429\ V_{11}[V^{+}]_{12}\ +\ 0.0492\ V_{12}[V^{+}]_{22}\ +\ 0.0230\ V_{13}[V^{+}]_{32}$
	2	$0.0492\ V_{11}[V^{+}]_{12}\ -\ 0.4429\ V_{12}[V^{+}]_{22}\ +\ 0.0230\ V_{13}[V^{+}]_{32}$
2	1	$1.2006\ V_{11}[V^{+}]_{12}\ -\ 0.0492\ V_{12}[V^{+}]_{22}\ -\ 0.0230\ V_{13}[V^{+}]_{32}$
	2	$-0.0492\ V_{11}[V^{+}]_{12}\ +\ 1.2006\ V_{12}[V^{+}]_{22}\ -\ 0.0230\ V_{13}[V^{+}]_{32}$

i	$D^{1}_{c,i}$
3	$-0.0240\ V_{11}[V^{+}]_{12}\ -\ 0.0240\ V_{12}[V^{+}]_{22}\ -\ 0.0089\ V_{13}[V^{+}]_{32}$
4	0
5	$0.0129\ V_{11}[V^{+}]_{12}\ +\ 0.0129\ V_{12}[V^{+}]_{22}\ +\ 0.0051\ V_{13}[V^{+}]_{32}$
6	$-0.0728\ V_{11}[V^{+}]_{12}\ -\ 0.0728\ V_{12}[V^{+}]_{22}\ -\ 0.0415\ V_{13}[V^{+}]_{32}$

TABLE 5. Coefficients to $C^{\theta\psi}_{b,i}$ and $D^{\theta}_{b,i}$ for the operator expansion of the Hamiltonian for penguin generating b decays given in eqn.(24b).

i	ψ	$C^{\theta\psi}_{b,i}$
1	1	$-0.2793[V^{+}]_{\theta 1} V_{13}$
	2	$-0.2793[V^{+}]_{\theta 2} V_{23}$
2	1	$1.1236[V^{+}]_{\theta 1} V_{13}$
	2	$1.1236[V^{+}]_{\theta 2} V_{23}$

i	$D^{\theta}_{b,i}$
3	$0.0127\,[V^{+}]_{\theta 1} V_{13} + 0.0127\,[V^{+}]_{\theta 2} V_{23} + 0.0031\,[V^{+}]_{\theta 3} V_{33}$
4	$-0.0284\,[V^{+}]_{\theta 1} V_{13} - 0.0284\,[V^{+}]_{\theta 2} V_{23} - 0.0056\,[V^{+}]_{\theta 3} V_{33}$
5	$0.0082\,[V^{+}]_{\theta 1} V_{13} + 0.0082\,[V^{+}]_{\theta 2} V_{23} + 0.0014\,[V^{+}]_{\theta 3} V_{33}$
6	$-0.0358\,[V^{+}]_{\theta 1} V_{13} - 0.0358\,[V^{+}]_{\theta 2} V_{23} - 0.0086\,[V^{+}]_{\theta 3} V_{33}$

TABLE 6. Coefficients $C_{t,i}^{\theta\psi}$ and $D_{t,i}^{\theta}$ of the operator expansion of the Hamiltonian of eqn.(24a) for penguin generating t decays. Note that in a 6 quark model the coefficients $D_{t,i}^{\theta}$ vanish for the relevant values $\theta = 1$ and 2 by unitarity of V.

i	$C_{t,i}^{\theta\psi}$
1	$-0.0605\ V_{\theta\psi}\ [V^+]_{\psi 3}$
2	$1.0218\ V_{\theta\psi}\ [V^+]_{\psi 3}$

i	$D_{t,i}^{\theta}$
3	$0.0024 \left\{V_{\theta 1}\ [V^+]_{13} + V_{\theta 2}\ [V^+]_{23} + V_{\theta 3}\ [V^+]_{33}\right\}$
4	$-0.0065 \left\{V_{\theta 1}\ [V^+]_{13} + V_{\theta 2}\ [V^+]_{23} + V_{\theta 3}\ [V^+]_{33}\right\}$
5	$0.0021 \left\{V_{\theta 1}\ [V^+]_{13} + V_{\theta 2}\ [V^+]_{23} + V_{\theta 3}\ [V^+]_{33}\right\}$
6	$-0.0069 \left\{V_{\theta 1}\ [V^+]_{13} + V_{\theta 2}\ [V^+]_{23} + V_{\theta 3}\ [V^+]_{33}\right\}$