BETA-DECAY, MAGNETIC MOMENTS, AND THE O(5) GROUP

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(*Received July 16, 1974*)

Assuming O(5) symmetry (pairing scheme of states for the system of protons and neutrons), the formulas for the probability of Gamov-Teller transition, the magnetic moment, and the probability of the (M1) transition were obtained as well as the relations between them. The comparison with the experimental data shows, except for few cases, quite good agreement in the region of light and very light nuclei.

1. *Introduction*

The symmetry properties of the system of nucleons, if properly foreseen, may reveal very hidden relations between quite different aspects of the system. The comparison of these relations with the actual experimental data can prove or disprove the symmetries assumed.

In what follows, we applied O(5) (orthogonal group) symmetry to the nucleons in nuclei to search for the probabilities of the Gamov-Teller transitions, the magnetic moments, and the relation between them. Moreover, in the Appendix, a similar treatment is given for the relation between the magnetic dipole and Gamov-Teller transitions.

2. *Mathematical preliminaries*

We take ten operators [1–4] as the generators of O(5) orthogonal group

$$A^+(M_T) = \tfrac{1}{2} \sum_{mm_1m_2} (\tfrac{1}{2}\, m_1\, \tfrac{1}{2}\, m_2 | 1 M_T)\, (-1)^{j-m} a^+_{jmm_1} a^+_{j-mm_2},$$

$$A(M_T) = [A^+(M_T)]^+,$$

$$T_+ = \sum_m a^+_{jm1/2} a_{jm-1/2}, \qquad T_- = [T_+]^+,$$

$$T_0 = \tfrac{1}{2} \sum_{mm_1} (-1)^{1/2-m_1} a^+_{jmm_1} a_{jmm_1},$$

$$H_1 = \tfrac{1}{2} [\sum_{mm_1} a^+_{jmm_1} a_{jmm_1} - (2j+1)] \equiv \tfrac{1}{2} [\hat{N} - (2j+1)], \tag{1}$$

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where $M_T = -1, 0, +1$ and $m_1, m_2 = \frac{1}{2}$ for protons and $-\frac{1}{2}$ for neutrons. T_+, T_-, T_0 are the ordinary isospin operators and $\hat{N}$ is the number of particles operator while H_1 and T_0 are the weight operators. A^+ is the pair creation operator with $J = 0$, $T = 1$. The state

$$|j^v T = t, M_T; JM\rangle \tag{2}$$

of v nucleons on the j level coupled to the total J and the total $T = t$, so called reduced isotopic spin, is, by definition, the state of seniority v if it does not comprise the pairs of nucleons coupled to $J = 0$ and $T = 1$. The same is given by the condition

$$A(M'_T)\,|j^v T = t, M_T; JM\rangle = 0 \tag{3}$$

for $M'_T = -1, 0, +1$. The irreducible representation (IR) of O(5) consists of all the states

$$[A^+(M_T^{(1)})A^+(M_T^{(2)}) \ldots A^+(M_T^{(r)})]^{T_r}_{M_{T_r}}|j^v T = t, M_T; JM\rangle. \tag{4}$$

The states (4) are characterized by the same seniority v and the reduced isotopic spin t. Instead of (vt), one takes usually the numbers $(\omega_1\omega_2)$ to mark the IR of O(5) with

$$\omega_1 = \tfrac{1}{2}(2j+1-v), \qquad \omega_2 = t. \tag{5}$$

To factorize the states within $(\omega_1\omega_2)$ one need four quantum numbers. Three of them are of a precise physical meaning. They are H_1, T and T_0. The fourth physical quantum number is, so far, unkown [5–10]. Fortunately, the physically important representations do not need the fourth number.

In general, the states of IR in O(5) scheme read

$$|(\omega_1\omega_2)H_1\beta TM_T\rangle, \tag{6}$$

where β is the fourth number. To fully factorize the states on the j-level one must add Sp$(2j+1)$ group numbers which comprise also the (JM). Then we get the states

$$|(\omega_1\omega_2)H_1\beta TM_T\alpha JM\rangle, \tag{7}$$

where α means the set of additional number needed for the classification within Sp$(2j+1)$.

The irreducible tensor operator in O(5) scheme is denoted by

$$T^{(\omega_1\omega_2)}_{H_1\beta TM_T}, \tag{8}$$

where the quantum numbers $(H_1\beta TM_T)$ take on all of the values allowed by the IR with $(\omega_1\omega_2)$.

It can be shown that the operators

$$U(JM; TM_T) \equiv \sum_{m_1m_2m_3m_4} (jm_1jm_2|JM)(\tfrac{1}{2}\,m_3\,\tfrac{1}{2}\,m_4|TM_T)a^+_{jm_1m_3}a_{j-m_2-m_4} \tag{9}$$

used in the further part of the work have a simple tensor character in O(5). Namely

$$\sqrt{2}\, U(JM;\, TM_T) = T^{(11)}_{0TM_T} \quad \text{for } J - \text{even},$$

$$-\sqrt{2}\, U(JM;\, 1M_T) = T^{(10)}_{01M_T} \quad \text{for } J - \text{odd},$$

$$\sqrt{2}\, U(JM;\, 00) = T^{(00)}_{(000)} \quad \text{for } J - \text{odd}, \tag{10}$$

where the quantum number β is not needed for the simple representations.

The sum over particles of one-particle operator of rank k in the angular momentum space and of rank T in the isospin space

$$F^{kT}_{qM_T} = \sum_{i=1}^{n} f(i)^{kT}_{qM_T} \tag{11}$$

can also be expressed as

$$F^{kT}_{qM_T} = \sum_{m_1 m_2 m_3 m_4} \langle jm_2\, \tfrac{1}{2}\, m_4 | f^{kT}_{qM_T} | jm_1\, \tfrac{1}{2}\, m_3 \rangle a^{+}_{jm_2 m_4} a_{jm_1 m_3}. \tag{12}$$

After the transformations and with the help of the Wigner-Eckart theorem one can obtain

$$F^{kT}_{qM_T} = \{(2k+1)(2T+1)\}^{-1/2} \langle j\, \tfrac{1}{2} \| f^{kT} \| j\, \tfrac{1}{2} \rangle U(kq;\, TM_T), \tag{13}$$

where the reduced matrix element is twice reduced: in the angular momentum space and in the isospin space.

On the other hand, the matrix element of the O(5) tensor operator can be expressed as follows.

$$\langle (\omega_1\omega_2) H_1 \beta T M_T | T^{(\omega''_1 \omega''_2)}_{H''_1 \beta'' T'' M''_T} | (\omega'_1 \omega'_2) H'_1 \beta' T' M'_T \rangle$$

$$= \sum_{\varrho} ((\omega'_1\omega'_2) H' \beta' T' M'_T;\, (\omega''_1\omega''_2) H''_1 \beta'' T'' M''_T | (\omega_1\omega_2) H_1 \beta T M_T)_{\varrho}$$

$$\times \langle (\omega_1\omega_2) \| T^{(\omega_1''\omega_2'')} \| (\omega'_1\omega'_2) \rangle_{\varrho}, \tag{14}$$

where the index ϱ distinguishes the same representations $(\omega_1\omega_2)$ which appear in the reduction of the Kronecker product $(\omega'_1\omega'_2) \otimes (\omega''_1\omega''_2)$. The Wigner coefficients

$$((\omega'_1\omega'_2) H'_1 \beta' T' M'_T;\, (\omega''_1\omega''_2) H''_1 \beta'' T'' M''_T | (\omega_1\omega_2) H_1 \beta T M_T) \tag{15}$$

are defined in the usual way

$$|[(\omega'_1\omega'_2)(\omega''_1\omega''_2)]\, (\omega_1\omega_2)\varrho;\, H_1 \beta T M_T\rangle$$

$$= \sum_{H'_1 \beta' T' M'_T H''_1 \beta'' T'' M''_T} ((\omega'_1\omega'_2) H'_1 \beta' T' M'_T;\, (\omega''_1\omega''_2) H''_1 \beta'' T'' M''_T | (\omega_1\omega_2) H_1 \beta T M_T)_{\varrho}$$

$$\times |(\omega'_1\omega'_2) H'_1 \beta' T' M'_T\rangle\, |(\omega''_1\omega''_2) H''_1 \beta'' T'' M''_T\rangle. \tag{16}$$

The Wigner coefficients can be also written as the product of the Clebsch-Gordan isospin coefficients and the "reduced" Wigner coefficients:

$$((\omega_1'\omega_2')H_1'\beta'T'M_T'; (\omega_1''\omega_2'')H_2''\beta''T''M_T''|(\omega_1\omega_2)H_1\beta TM_T)_\varrho$$
$$\equiv (T'M_T'T''M_T''|TM_T)\times((\omega_1'\omega_2')H_1'\beta'T'; (\omega_1''\omega_2'')H_1''\beta''T''\|(\omega_1\omega_2)H_1\beta T)_\varrho. \tag{17}$$

The reduced Wigner coefficients of O(5), which will be needed, were tabulated in the papers [6, 8, 12].

3. *The Gamov-Teller transition*

The probability of the Gamov-Teller β transition, for which $\Delta J = \pm 1, 0$, is given by [13]

$$B(J'T'M_T' \to J''T''M_T'') =$$
$$= 2g_{\mathrm{GT}}^2 \sum_{\mu M''} |\langle J''M''T''M_T''| \sum_k \sigma_\mu(k)\tau_{\pm 1}(k)|J'M'T'M_T'\rangle|^2, \tag{18}$$

where

$$\sigma_{\pm 1} = \mp\sqrt{\tfrac{1}{2}}\,(\sigma_x \pm \sigma_y), \qquad \sigma_0 = \sigma_z, \qquad \tau_\pm = \mp\sqrt{\tfrac{1}{2}}\,(\tau_x \pm \tau_y)$$

are the isospin one-particle tensor operators and the spin (Pauli) tensor operators of the rank 1 in both spaces. The sum over k is the sum over particles of the system. τ_{+1} operator is responsible for β^+ disintegration and τ_{-1} — for β^- disintegration.

Following (13) we get

$$B(J'T'M_T'' \to J''T''M_T'') = \tfrac{2}{9}\, g_{\mathrm{GT}}|\langle \tfrac{1}{2}\,\|\tau\|\,\tfrac{1}{2}\rangle|^2|\langle j\,\|\sigma\|\,j\rangle|^2$$
$$\times \sum_{\mu M''} |\langle J''M''T''M_T''|U(1\mu;\, 1\nu)|J'M'T'M_T'\rangle|^2$$
$$= \tfrac{1}{3}\, g_{\mathrm{GT}}^2(2J'+1)^{-1}|\langle j\|\sigma\|j\rangle|^2|\langle J''T''M_T''\,\|U(1;\, 1\nu)\|J'T'M_T'\rangle|^2, \tag{19}$$

where $\nu = \pm 1$.

Now, assuming that the nuclear states are of O(5) states (7) we get

$$B[(\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J' \to (\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J'']$$
$$= \tfrac{1}{3}\, g_{\mathrm{GT}}^2(2J^+ +1)^{-1}|\langle j\,\|\sigma\|\,j\rangle|^2$$
$$\times|\langle(\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J''\,\|U(1;\, 1\nu)\|(\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J'\rangle|^2. \tag{20}$$

We can make, in a standard method for O(5), the reduction from the state with n-particles ($H_1 = n/2-(j+\frac{1}{2})$) to the state of v-particles (v-seniority number) making use of the tensor character of $U(1;\, 1\nu)$ (see (10)). We get

$$\langle(\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J''\,\|U(1;\, 1\nu)\|(\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J'\rangle$$
$$= (2\bar{T}''+1)^{-1/2}(T'M_T'1\nu|T''M_T'')\,\frac{((\omega_1'\omega_2')H_1\beta'T';\, (10)01\,\|(\omega_1''\omega_2'')H_1\beta''T'')}{((\omega_1'\omega_2')H_{\bar v}\bar{T};\, (10)01\,\|(\omega_1''\omega_2'')H_{\bar v}\bar{T}'')}$$
$$\times\langle(\omega_1''\omega_2'')H_{\bar v}\bar{T}''\alpha''J''\,\|U(1;\, 1)\|(\omega_1'\omega_2')H_{\bar v}T'\alpha'J'\rangle, \tag{21}$$

where $H_{\bar{v}} = \bar{v}/2-(j+\frac{1}{2})$, $\bar{v} = \max(v', v'')$, and $U(1; 1)$ is reduced in $O_J(3)\otimes O_T(3)$. From the equations (20) and (21) we get the final general form for the transition probability

$$B[(\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J' \rightarrow (\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J'']$$
$$= \tfrac{1}{3}\, g_{GT}^2(2J'+1)^{-1}(2\bar{T}''+1)^{-1}|\langle j\|\sigma\|j\rangle|^2(T'M_T'1\nu|T''M_T'')^2$$
$$\times \frac{((\omega_1'\omega_2')H_1\beta'T';(10)01\|(\omega_1''\omega_2'')H_1\beta''T'')^2}{((\omega_1'\omega_2')H_{\bar{v}}\bar{T}';(10)01\|(\omega_1''\omega_2'')H_{\bar{v}}\bar{T}'')^2}$$
$$\times |\langle(\omega_1''\omega_2'')H_{\bar{v}}\bar{T}''\alpha''J''\|U(1;1)\|(\omega_1'\omega_2')H_{\bar{v}}\bar{T}'\alpha'J'\rangle|^2. \tag{22}$$

The selecting rule, imposed on the transition $(\omega_1'\omega_2') \rightarrow (\omega_1''\omega_2'')$, is given by the Kronecker product

$$(\omega_1, \omega_2)\otimes(1, 0) = (\omega_1\omega_2)\oplus(\omega_1+1, \omega_2)\oplus(\omega_1-1, \omega_2)\oplus(\omega_1, \omega_2+1)\oplus(\omega_1, \omega_2-1). \tag{23}$$

Taking the states with the seniority $\bar{v} \leqslant 2$ only, we obtain, with the help of (5) and (23), the following allowed transitions $(\omega_1'\omega_2') \rightarrow (\omega_1''\omega_2'')$ on the j-level:

$$\begin{array}{llll}
(j, \quad \frac{1}{2}) \rightarrow (j, \quad \frac{1}{2}); & v = 1 \rightarrow 1 & t = \frac{1}{2} \rightarrow \frac{1}{2} & \Delta J = 0,\\
(j-\frac{1}{2}, 0) \rightarrow (j-\frac{1}{2}, 0): & v = 2 \rightarrow 2 & t = 0 \rightarrow 0 & \Delta J = 0,\\
(j-\frac{1}{2}, 0) \rightarrow (j+\frac{1}{2}, 0): & v = 2 \rightarrow 0 & t = 0 \rightarrow 0 & \Delta J = -1,\\
(j-\frac{1}{2}, 0) \rightarrow (j-\frac{1}{2}, 1): & v = 2 \rightarrow 2 & t = 0 \rightarrow 1 & \Delta J = \pm 1,\\
(j-\frac{1}{2}, 1) \rightarrow (j-\frac{1}{2}, 1): & v = 2 \rightarrow 2 & t = 1 \rightarrow 1 & \Delta J = 0.
\end{array} \tag{24}$$

For $v = 1$ we have $J' = J'' = j$, $\bar{T}' = \bar{T}'' = \frac{1}{2}$; the reduced matrix element in (22) is equal to 3 and

$$\left|\left\langle\left(l\frac{1}{2}\right)j\|\sigma\|\left(l\frac{1}{2}\right)j\right\rangle\right|^2 = \frac{2j+1}{j(j+1)} \times \left[j(j+1)-l(l+1)+\frac{3}{4}\right]^2. \tag{25}$$

Then from (22) we get

$$B[(j, \tfrac{1}{2})H_1T'M_T'j \rightarrow (j, \tfrac{1}{2})H_1T''M_T''j]$$
$$= \frac{3}{2}\, g_{GT}^2 \frac{j+2}{j(j+1)^2} \times \left[j(j+1)-l(l+1)+\frac{3}{4}\right]^2 (T'M_T'1\nu|T''M_T'')^2$$
$$\times ((j, \tfrac{1}{2})H_1T';(10)01\|(j, \tfrac{1}{2})H_1T'')^2, \tag{26}$$

in which we have used the value of the Wigner coefficient

$$\left(\left(j, \frac{1}{2}\right)H_1 = -j, \frac{1}{2};(10)01\|\left(j, \frac{1}{2}\right)H_1 = -j, \frac{1}{2}\right) = \sqrt{\frac{j+1}{j+2}}. \tag{27}$$

For the last four transitions with $v = 2$ in (24), the reduced matrix element is

$$\langle H_{\bar{v}}\bar{T}''J''\|U(1;1)\|H_{\bar{v}}\bar{T}'J'\rangle$$

$$= -6[(2J'+1)(2J''+1)(2\bar{T}'+1)(2\bar{T}''+1)]^{1/2}\begin{Bmatrix} J' & J'' & 1 \\ j & j & j \end{Bmatrix}\begin{Bmatrix} \bar{T}' & \bar{T}'' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}. \tag{28}$$

Then we get

$$B[(\omega_1'\omega_2')H_1\beta'T'M_T'J' \rightarrow (\omega_1''\omega_2'')H_1\beta''T''M_T''J'']$$

$$= 12g_{\mathrm{GT}}^2\frac{(2j+1)(2J''+1)(2\bar{T}'+1)}{j(j+1)}\left[j(j+1)-l(l+1)+\frac{3}{4}\right]^2$$

$$\times(T'M_T'1v|T''M_T'')^2\begin{Bmatrix} J' & J'' & 1 \\ j & j & j \end{Bmatrix}^2\begin{Bmatrix} \bar{T}' & \bar{T}'' & 1 \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{Bmatrix}^2$$

$$\times\frac{((\omega_1'\omega_2')H_1\beta'T';(10)01\|(\omega_1''\omega_2'')H_1\beta''T'')^2}{((\omega_1'\omega_2')H_{\bar{v}}\bar{T};(10)01\|(\omega_1''\omega_2'')H_{\bar{v}}\bar{T}'')^2}, \tag{29}$$

where $\bar{v} = 2$ and $H_{\bar{v}} = -j+\frac{1}{2}$ (see (1)) for all the cases.

By the symmetry relations of the Wigner and Clebsch-Gordan coefficients the formula for the reversed transition is also obtained:

$$B[(\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J'' \rightarrow (\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J']$$

$$= \frac{2J'+1}{2J''+1}\cdot B[(\omega_1'\omega_2')H_1\beta'T'M_T'\alpha'J' \rightarrow (\omega_1''\omega_2'')H_1\beta''T''M_T''\alpha''J'']. \tag{30}$$

4. *Log ft*

In general, the probability of an allowed β transition is composed of the Gamov-Teller and Fermi transitions. For the last one we have the simple formula

$$A[JTM_T \rightarrow JTM_T\pm1] = g_{\mathrm{F}}^2|\langle JMTM_T\pm1|T_\pm|JMTM_T\rangle|^2$$
$$= g_{\mathrm{F}}^2[T(T+1)-M_T(M_T\pm1)]^2 \tag{31}$$

as the transition operator is the tensor only in $O_T(3)$ space.

The non-relativistic theory of β transition gives the ft:

$$ft = \frac{\Gamma}{\bar{A}+R\bar{B}}, \tag{32}$$

where Γ is constant, $R = g_{\mathrm{GT}}^2/g_{\mathrm{F}}^2$ and

$$\bar{B} = B/g_{\mathrm{GT}}^2; \quad \bar{A} = A/g_{\mathrm{F}}^2$$

are given by (26), (29) and (31). The values Γ and R are determined by the experimental values for the β disintegration of the free neutron. They are (within 5% error):

$$\Gamma = (6.3\pm0.3)\cdot10^{-3};\ R = 1.39\pm0.04. \tag{33}$$

TABLE I

The theoretical values of log ft assuming O(5) nuclear states, and experimental ones taken from [11] for nuclei with $A \leqslant 45$

nlj level	States: initial (JTM_T)	States: final (JTM_T)	Transitions $(\omega_1\omega_2) \to (\omega_1'\omega_2')$	log ft: theor.	log ft: experim.
$1p_{3/2}$	^{6}He $(0\ 1\ 1)$	^{6}Li $(1\ 0\ 0)$	$(2, 0) \to (1, 0)$	3.13	2.95
$1p_{3/2}$	^{7}Be $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{7}Li $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{3}\ \frac{1}{3}) \to (\frac{3}{3}\ \frac{1}{3})$	3.27	3.3
$1p_{3/2}$	^{8}Li $(2\ 1\ 1)$	^{8}Be $(2\ 0\ 0)$	$(1, 1) \to (1, 1)$	3.53	5.6
$1p_{3/2}$	^{8}Be $(2\ 1\ -1)$	^{8}Be $(2\ 0\ 0)$	$(1, 1) \to (1, 1)$	3.53	5.6
$1p_{3/2}$	^{8}B $(2\ 1\ -1)$	^{8}Be $(2\ 1\ 0$	$(1, 1) \to (1, 1)$	3.50	2.9
$1p_{3/2}$	^{9}Li $(\frac{3}{2}\ \frac{3}{2}\ \frac{3}{2})$	^{9}Be $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	4.00	5.5
$1p_{3/2}$	^{10}C $(0\ 1\ -1)$	^{10}B $(1\ 0\ 0)$	$(2, 0) \to (1, 0)$	3.13	3.0
$1p_{3/2}$	^{11}C $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{11}B $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	3.28	3.6
$1p_{1/2}$	^{13}N $(\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{13}C $(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{1}{2}\ \frac{1}{2}) \to (\frac{1}{2}\ \frac{1}{2})$	3.64	3.65
$1p_{1/2}$	^{15}O $(\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{15}N $(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{1}{2}\ \frac{1}{2}) \to (\frac{1}{2}\ \frac{1}{2})$	3.64	3.65
$1d_{5/2}$	^{17}F $(\frac{5}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{17}O $(\frac{5}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{5}{2}\ \frac{1}{2}) \to (\frac{5}{2}\ \frac{1}{2})$	3.33	3.36
$1d_{5/2}$	^{18}F $(1\ 0\ 0)$	^{18}O $(0\ 1\ 1)$	$(2, 0) \to (3, 0)$	3.68	3.62
$1d_{5/2}$	^{21}Mg $(\frac{5}{2}\ \frac{3}{2}\ -\frac{3}{2})$	^{21}Na $(\frac{5}{2}\ \frac{3}{2}\ -\frac{1}{2})$	$(\frac{5}{2}\ \frac{1}{2}) \to (\frac{5}{2}\ \frac{1}{2})$	3.25	2.9
$1d_{5/2}$	^{24}Al $(4\ 1\ -1)$	^{24}Mg $(4\ 1\ 0)$	$(2, 1) \to (2, 1)$	3.48	3.4
$1d_{5/2}$	^{25}Na $(\frac{5}{2}\ \frac{3}{2}\ \frac{3}{2})$	^{25}Mg $(\frac{5}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{5}{2}\ \frac{1}{2}) \to (\frac{5}{2}\ \frac{1}{2})$	3.93	5.3
$1d_{5/2}$	^{26}Si $(0\ 1\ -1)$	^{26}Al $(1\ 0\ 0)$	$(3, 0) \to (2, 0)$	3.20	3.35
$1d_{5/2}$	^{27}Si $(\frac{5}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{27}Al $(\frac{5}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{5}{2}\ \frac{1}{2}) \to (\frac{5}{2}\ \frac{1}{2})$	3.50	3.50
$2s_{1/2}$	^{29}P $(\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{29}Si $(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{1}{2}\ \frac{1}{2}) \to (\frac{1}{2}\ \frac{1}{2})$	3.08	3.7
$2s_{1/2}$	^{31}S $(\frac{1}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{31}P $(\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{1}{2}\ \frac{1}{2}) \to (\frac{1}{2}\ \frac{1}{2})$	3.08	3.7
$1d_{3/2}$	^{33}Cl $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{33}S $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	3.53	3.7
$1d_{3/2}$	^{34}Cl $(3\ 0\ 0)$	^{34}S $(2\ 1\ 1)$	$(1, 0) \to (1, 1)$	4.27	4.9
$1d_{3/2}$	^{34}Ar $(0\ 1\ -1)$	^{34}Cl $(1\ 0\ 0)$	$(2, 0) \to (1, 0)$	3.52	5.2
$1d_{3/2}$	^{35}S $(\frac{3}{2}\ \frac{3}{2}\ \frac{3}{2})$	^{35}Cl $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	4.45	5.0
$1d_{3/2}$	^{35}Ar $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{35}Cl $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	3.64	3.8
$1d_{3/2}$	^{36}K $(2\ 1\ -1)$	^{36}Ar $(2\ 1\ 0)$	$(1, 1) \to (1, 1)$	3.5	3.6
$1d_{3/2}$	^{37}Ar $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	^{37}Cl $(\frac{3}{2}\ \frac{3}{2}\ \frac{3}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	4.45	5.0
$1d_{3/2}$	^{37}K $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{37}Ar $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	3.64	3.6
$1d_{3/2}$	^{37}Ca $(\frac{3}{2}\ \frac{3}{2}\ -\frac{3}{2})$	^{37}K $(\frac{3}{2}\ \frac{3}{2}\ -\frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	4.45	5.0
$1d_{3/2}$	^{38}K $(3\ 0\ 0)$	^{38}Ar $(2\ 1\ 1)$	$(1, 0) \to (1, 1)$	4.27	5.0
$1d_{3/2}$	^{39}Ca $(\frac{3}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{39}K $(\frac{3}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{3}{2}\ \frac{1}{2}) \to (\frac{3}{2}\ \frac{1}{2})$	3.53	3.6
$1f_{7/2}$	^{41}Sc $(\frac{7}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{41}Ca $(\frac{7}{2}\ \frac{1}{2}\ \frac{1}{2})$	$(\frac{7}{2}\ \frac{1}{2}) \to (\frac{7}{2}\ \frac{1}{2})$	3.35	3.5
$1f_{7/2}$	^{43}Sc $(\frac{7}{2}\ \frac{1}{2}\ \frac{1}{2})$	^{43}Ca $(\frac{7}{2}\ \frac{3}{2}\ \frac{3}{2})$	$(\frac{7}{2}\ \frac{1}{2}) \to (\frac{7}{2}\ \frac{1}{2})$	3.90	5.1
$1f_{7/2}$	^{43}Ti $(\frac{7}{2}\ \frac{1}{2}\ -\frac{1}{2})$	^{43}Sc $(\frac{7}{2}\ \frac{3}{2}\ \frac{3}{2})$	$(\frac{7}{2}\ \frac{1}{2}) \to (\frac{7}{2}\ \frac{1}{2})$	3.61	3.5
$1f_{7/2}$	^{45}Ti $(\frac{7}{1}\ \frac{1}{2}\ \frac{1}{2})$	^{45}Sc $(\frac{7}{2}\ \frac{3}{2}\ \frac{3}{2})$	$(\frac{7}{2}\ \frac{1}{2}) \to (\frac{7}{2}\ \frac{1}{2})$	4.63	4.5

In Table I the experimental and theoretical values of $\log ft$ are given for 34 disintegrations of light nuclei with $A \leqslant 45$. For more than half of the disintegrations the accuracy of the theoretical values is quite satisfactory, for one fourth — fairly good, and for some 10 decays the agreement is rather poor. The comparison may be used as a check of how good is O(5) symmetry, in real nuclei. Moreover, the discrepancies of the experimental and theoretical results may be also used to determine the mixing configurations in O(5) symmetry numbers.

5. The relation between the magnetic moments and the probabilities of the Gamov-Teller transition

The dipole magnetic moment operator for the simple system of protons and neutrons reads

$$\hat{\mu}_z = \sum_i \{[\tfrac{1}{2}+t_z^{(i)}]\sigma_z^{(i)}\mu_n+[\tfrac{1}{2}-t_z^{(i)}]\,[l_z^{(i)}+\sigma_z^{(i)}]\mu_p\}$$

$$= \tfrac{1}{2}\sum_i j_z^{(i)}+\tfrac{1}{2}\,a\sum_i \sigma_z^{(i)}-\sum_i t_z^{(i)}[j_z^{(i)}+\mathfrak{b}\sigma_z^{(i)}], \tag{34}$$

where $a = \mu_p+\mu_n-\frac{1}{2}$; $b = \mu_p-\mu_n-\frac{1}{2}$, and $t_z^{(i)}, \tau_z^{(i)}, l_z^{(i)}, j_z^{(i)}$ are the isospin, the spin (Pauli), the orbital momentum, and the total angular momentum operators for the i-th particle. In second quantization, using (9) operators, we can express $\hat{\mu}_z$ as

$$\hat{\mu}_z = \frac{1}{2}\left[1+a\frac{\langle j\|\sigma\|j\rangle}{\sqrt{j(j+1)(2j+1)}}\right]J_z+\sqrt{\frac{1}{6}}\,[\langle j\|j\|j\rangle+\mathfrak{b}\langle j\|\sigma\|j\rangle]U(10;10), \tag{35}$$

where

$$J_z = \sum_{m_1m_2} m_1 a^+_{jm_1\tau m_2}a_{jm_1\tau m_2} \tag{36}$$

is the z-component of the total angular momentum of a system.

To calculate the matrix element of $\hat{\mu}_z$ operator one only needs to calculate the matrix element

$$\langle JM = J|U(1,0;10)|JM = J\rangle$$

with the help of O(5) tensor character (1, 0). Performing the reduction in the whole O(5) we get in $(\omega_1\omega_2)$ basis

$$\langle\mu\rangle \equiv \mu[(\omega_1\omega_2)H_1\beta TM_T\alpha JM = J] = \frac{1}{2}\left[1+a\cdot\frac{\langle j\|\sigma\|j\rangle}{\sqrt{j(j+1)(2j+1)}}\right]\cdot J$$

$$-\frac{1}{6}[\langle j\|j\|j\rangle\cdot\mathfrak{b}\langle j\|\sigma\|j\rangle]\cdot\frac{J}{\sqrt{J(J+1)(2J+1)}}$$

$$\times(TM_T10|TM_T)\,((\omega_1\omega_2)H_1\beta T;(10)01\|(\omega_1\omega_2)H_1\beta T)\,\langle(\omega_1\omega_2)\alpha J\|U(1;)\|(\omega_1\omega_2)\alpha J\rangle. \tag{37}$$

On the other hand, starting with (20) we can perform further reduction in O(5) and get

$$\bar{B}[(\omega_1\omega_2)H_1\beta'T'M_T'\alpha J \to (\omega_1\omega_2)H_1\beta''T''M_T''\alpha J]$$
$$= \tfrac{1}{3}(2J+1)^{-1}|\langle j\|\sigma\|j\rangle|^2(T'M_T'1\nu|T''M_T'')^2$$
$$\times((\omega_1\omega_2)H_1\beta'T';(10)01\|(\omega_1\omega_2)H_1\beta''T'')^2$$
$$\times\langle(\omega_1\omega_2)\alpha J\|U(1;)\|(\omega_1\omega_2)\alpha J\rangle^2. \quad (38)$$

The elimination of the reduced matrix element of U operator in (37) and (38) is followed by the final relation between $\bar{B}$ and μ:

$$\bar{B} = \frac{J+1}{2J}\cdot\frac{[2(l+\frac{1}{2})\mu-(l+\frac{1}{2}\pm a)]^2}{(l+\frac{1}{2})^2(l+\frac{1}{2}+\mathfrak{b})^2}\cdot\frac{(T'M_T'1\nu|T''M_T'')^2}{(TM_T10|TM_T)^2}$$
$$\times\frac{((\omega_1\omega_2)H_1\beta'T';(10)01\|(\omega_1\omega_2)H_1\beta''T'')^2}{((\omega_1\omega_2)H_1'\beta T;(10)01\|(\omega_1\omega_2)H_1'\beta T)^2}, \quad (39)$$

where the signs $\pm a$ are for $j = l\pm\frac{1}{2}$ couplings, and $\nu = \pm 1$ is for β^+ and β^- transitions, respectively. Substituting in (39):

$$T' = T'' = T = \tfrac{1}{2}, \quad \beta' = \beta'' = \beta, \quad H_1' = H_1$$

we get the Goeppert-Mayer and Jensen relation [14] for mirror nuclei

$$\bar{B} = \frac{J+1}{J}\cdot\frac{[2(l+\frac{1}{2})\mu-(l+\frac{1}{2}\pm a)]^2}{(l+\frac{1}{2})^2(l+\frac{1}{2}+\mathfrak{b})^2}. \quad (40)$$

Relation (39) is then the generalization of the Goeppert-Mayer and Jensen formulae (40). It connects the magnetic moment of nuclei in the state belonging to a given irreducible representation $(\omega_1\omega_2)$ of O(5) with the probabilities of the Gamov-Teller transition between the different states of the same representation $(\omega_1\omega_2)$.

TABLE II

The example of the relation between the probabilities of the Gamov-Teller transitions and magnetic moments. The similar values of log ft were obtained from quite different (column 2) magnetic moments. The nuclear states were considered as the states $(\omega_1\omega_2) = (\frac{3}{2}\,\frac{1}{2})$ of O(5) representation

$(\omega_1\omega_2) = (\frac{3}{2}, \frac{1}{2})$; $l = 2$; $j = \frac{3}{2}$; $J = \frac{3}{2}$

Nucleus (H_1TM_T)	Magnetic moments	Transitions $(H_1TM_T) \to (H_1TM_T)$	log ft calcul. from [39] with exp. μ	log ft exp.
^{35}Cl $(-\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})$	0.8218	^{35}S $\to$ ^{35}Cl	5.20	
^{35}S $(-\frac{1}{2}\,\frac{3}{2}\,\frac{3}{2})$	1.00	$(-\frac{1}{2}\,\frac{3}{2}\,\frac{3}{2}) \to (-\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})$	4.65	
^{37}Ar $(\frac{1}{2}\,\frac{3}{2}\,\frac{1}{2})$	0.95	or	4.61	5.0
^{37}Cl $(\frac{1}{2}\,\frac{3}{2}\,\frac{3}{2})$	0.6841	^{37}Ca $\to$ ^{39}K	6.40	
^{39}K $(\frac{3}{2}\,\frac{1}{2}\,\frac{1}{2})$	0.3914	$(\frac{1}{2}\,\frac{3}{2}\,-\frac{3}{2}) \to (\frac{1}{2}\,\frac{1}{2}\,-\frac{1}{2})$	4.99	

The validity of O(5) symmetry can be then checked by (39). Several nuclei belonging to a given $(\omega_1 \omega_2)$ supply different magnetic moments which can be put in (39) to reproduce $\bar{B}$ transition probability between the two nuclei of the same $(\omega_1 \omega_2)$ (see Table II). It is seen, for example, from Table II that the magnetic moments differing by the factor 2.5(^{35}S; ^{39}K) can reproduce within O(5) symmetry almost the same $\bar{B}$ as it ought to be.

APPENDIX

Magnetic dipole and the Gamov-Teller transitions

The probability of L-pole magnetic transition is given by [15]:

$$B(\mathrm{M}L) = \left(\frac{e\hbar}{2m_{\mathrm{p}}c}\right)^2 \sum_{MM_2} |\langle J_2 M_2 T_2 M_{T_2}| \sum_i (\nabla r^l Y^*_{LM})_i$$

$$\times \left\{\left[\frac{1}{2}(\mu_{\mathrm{n}}+\mu_{\mathrm{p}})\vec{\sigma}^{(i)} + \frac{1}{L+1}\vec{l}^{(i)}\right] + 2\tau_3^{(i)}\left[\frac{1}{2}(\mu_{\mathrm{n}}-\mu_{p})\vec{\sigma}^{(i)} - \frac{1}{L+1}\vec{l}^{(i)}\right]\right\} |J_1 M_T T_1 M_{T_1}\rangle|^2. \tag{1A}$$

The transition operator in (1A) consists of two parts: isoscalar and isovector. The relative contribution of the first part to the second is of the order:

$$\frac{(\mu_{\mathrm{n}}+\mu_{\mathrm{p}}-\frac{1}{2})^2}{(\mu_{\mathrm{n}}-\mu_{\mathrm{p}}+\frac{1}{2})^2} = \left(\frac{0.38}{4.20}\right)^2 = 0.082.$$

In what follows we take only the isovector part of the (1A) and writing it for (M1) transition in second quantization form we get

$$B(\mathrm{M}1) = \frac{1}{8\pi}\cdot\left(\frac{e\hbar}{2m_{\mathrm{p}}c}\right)^2 (2J_1+1)^{-1}\{(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\langle j\|\sigma\|j\rangle + \langle j\|l\|j\rangle\}^2$$

$$\times |\langle J_2 T_2 M_{T_2}\|U(1;\,10)\|J_1 T_1 M_{T_1}\rangle|^2. \tag{2A}$$

$U(1\mathrm{M};\,10)$ operator is of the type (10): $T^{(10)}_{010}$ in O(5). The Wigner-Eckart theorem for O(5) gives then

$$B(\mathrm{M}1) = \frac{1}{8\pi}\cdot\left(\frac{e\hbar}{2m_{\mathrm{p}}c}\right)^2 (2J_1+1)^{-1}(T_2 M_{T_2} 10|T_1 M_{T_1})^2\{(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\langle j\|\sigma\|j\rangle + \langle j\|l\|j\rangle\}^2$$

$$\times((\omega_1'\omega_2')H_1\beta_1 T_1;\,(10)01\|(\omega_1''\omega_2'')H_1\beta_2 T_2)^2$$

$$\times|\langle(\omega_1''\omega_2'')\alpha_2 J_2\|U(1;)\|(\omega_1'\omega_2')\alpha_1 J_1\rangle|^2. \tag{3A}$$

Comparing it with (38) for $\bar{B}$, for $(\omega_1'\omega_2'\alpha_1 J_1)$ and $(\omega_1''\omega_2''\alpha_2 J_2)$ states we get the relation

$$\frac{B(\mathrm{M1})}{\bar{B}} = \frac{3}{8\pi}\cdot\left(\frac{eh}{2m_{\mathrm{p}}c}\right)^2\cdot(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})^2\cdot\frac{(T_1 M_{T_1} 10|T_2 M_{T_2})^2}{(T' M_T' 1\nu|T'' M_T'')^2}$$

$$\times\left\{1+\frac{1}{\mu_{\mathrm{p}}-\mu_{\mathrm{n}}}\cdot\frac{\langle j\|l\|j\rangle}{\langle j\|\sigma\|j\rangle}\right\}^2\cdot\frac{((\omega_1'\omega_2')H_1\beta_1 T_1;(10)01\|(\omega_1''\omega_2'')H_1\beta_2 T_2)^2}{((\omega_1'\omega_2')H_1'\beta' T';(10)01\|(\omega_1''\omega_2'')H_1'\beta'' T'')^2}. \quad (4A)$$

Relation (4A), valid for O(5) symmetry, is taken for two pairs of transition nuclei for which $(\omega_1'\omega_2'\alpha_1 J_1)$ quantum numbers are the same for initial states in both transitions and $(\omega_1''\omega_2''\alpha_2 J_2)$ are common for final states. Relation (4A) is analogical to the generalized Goeppert-Mayer and Jensen formula (39).

Substituting in (4A):

$$H_1 = H_1', \quad \beta_1 = \beta', \quad \beta_2 = \beta''; \quad T_1 = T', \quad T_2 = T''$$

we get the Kurath [16] relation:

$$\frac{B(\mathrm{M1})}{\bar{B}} = \frac{3}{8\pi}\cdot\left(\frac{eh}{2m_{\mathrm{p}}c}\right)^2(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\cdot\frac{(T' M_{T_1} 10|T'' M_{T_2})^2}{(T' M_T' 1\nu|T'' M_T'')^2}$$

$$\times\left\{1+\frac{1}{(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})}\cdot\frac{\langle j\|l\|j\rangle}{\langle j\|\sigma\|j\rangle}\right\}^2. \quad (5A)$$

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