

# From Sakata Model to Goldberg-Ne'eman Quarks and Nambu QCD Phenomenology and “Right” and “Wrong” experiments

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The basic theoretical milestones were the Sakata SU(3) symmetry, the Goldberg-Ne'eman composite model with SU(3) triplets having baryon number (1/3) and the Nambu color gauge Lagrangian. The transition was led in right and wrong directions by experiments interpreted by phenomenology. A “good” experiment on  $\bar{p}p$  annihilation at rest showed that the Sakata model predictions disagreed with experiment. A “bad” experiment prevented the use of the Goldberg-Ne'eman triplet model to predict the existence and masses of the of the  $\Xi^*$  and  $\Omega^-$ . More “good” experiments revealed the existence and mass of the  $\Xi^*$  and the  $\Omega^-$  and the absence of positive strangeness baryon resonances, thus confirming the “tenfold way”. Further “good experiments” revealed the existence of the vector meson nonet, SU(3) breaking with singlet-octet mixing and the suppression of the  $\phi \rightarrow \rho\pi$  decay. These led to the quark triplet model. The paradox of peculiar statistics then arose as the  $\Delta^{++}$  and  $\Omega^-$  contained three identical spin-1/2 fermions coupled symmetrically to spin (3/2). This led to color and the Nambu QCD. The book “Lie Groups for Pedestrians” used the Sakata model with the name “sakaton” for the  $pn\Lambda$  triplet to teach the algebra of SU(3) to particle physicists in the U.S. and Europe who knew no group theory. The Sakata model had a renaissance in hypernuclear physics in the 1970's.

## §1. The Role of experiments, right and wrong

Physics is an experimental science. The models and ideas that remain are determined by experimental tests. Right experiments can disprove wrong models. Wrong experiments can lead theorists astray. Both of these occurred in the transition from the Sakata model<sup>1)-3)</sup> to the Goldberg-Ne'eman-Gell-Mann-Zweig quark model.<sup>4)</sup>

The same SU(3) “unitary symmetry” group was used both in the Sakata model and in the Gell-Mann-Ne'eman octet model called the “Eightfold Way”.

SU(3) is a natural symmetry of the Sakata model which is built on a fundamental triplet. But there is no obvious fundamental triplet in the eightfold way. Goldberg and Ne'eman showed how to build the baryon octet from fundamental triplets.<sup>4)</sup>

A “Right Experiment”<sup>5)</sup> showed that the Sakata model disagreed with experiment. The reaction  $\bar{p}p \rightarrow K_L K_S$  forbidden by Sakata Model was found.

A “Wrong Experiment” showed that Goldberg-Ne'eman triplets disagreed with

experiment. The decay  $\Sigma^* \rightarrow \Sigma\pi$  allowed by G-N was not found.<sup>6)</sup>

$$\frac{BR[\Sigma(1385) \rightarrow \Sigma\pi]}{BR[\Sigma(1385) \rightarrow \Lambda\pi]} = (4 \pm 4)\% \quad (1.1)$$

This led to a wrong selection rule<sup>6)</sup> forbidding  $\Sigma^* \rightarrow \Sigma\pi$  and requiring the  $\Sigma^*$  to be in an exotic 27 - dimensional representation of SU(3).

## §2. From Sakata to Goldberg-Ne'eman, Gell-Mann, Zweig, Nambu

### 2.1. Experiments and phenomenology

The impact of the Sakata model in the period before 1964 has been described by Okun.<sup>7)</sup> This paper considers the period beginning in 1961, describes the impact of the Sakata model in the overlapping period 1961-64 from a very different perspective and continues on beyond.

The steps leading from the Sakata model to the quark model via the Goldberg-Ne'eman model and eventually to Nambu QCD were led by experiments interpreted by phenomenology.

The Goldberg-Ne'eman paper *anticipated* the quark model. All the basic physics of constructing the baryon octet from three triplets with baryon number (1/3) is in the paper. That the triplets must have fractional charge if they are identical is obvious to any graduate student who understands group theory. That the U(3) classification of mesons required nonets containing a singlet as well as an octet of SU(3) should also have been obvious. One might also have seen that the  $K - \pi$  mass difference required breaking  $U(3)$  to  $U(2) \otimes U(1)$  and what was later was called "ideal mixing" of the SU(3) singlet and octet vector mesons. But the paper was a bit ahead of its time. The experimental situation was not ready for its acceptance.

The wrong experiment (1.1) pushed theorists into classifying the  $\Delta(1238)$  and  $\Sigma(1385)$  in the 27-plet of SU(3). This pushed experimenters to search for their partners in the 27-plet, the  $K^+N$  resonances which were not found.

The discovery of the  $\Xi^*$  (1530) led Glashow and Sakurai<sup>8)</sup> to classify it together with the  $\Delta(1238)$  and  $\Sigma(1385)$  in "The Tenfold Way" and predict the existence of an isoscalar baryon with strangeness (-3) and a mass low enough to make it stable against strong decays. This was also noted by some others, including Gell-Mann and Ne'eman, who pointed it out informally at the ICHEP (Rochester) conference in Geneva in 1962. But the first published calculation was by Glashow and Sakurai who also did a detailed analysis with an estimate of its possible production in cosmic rays, and found a serious candidate event. To an unbiased observer, the credit for the prediction of the existence of the  $Z^-$ , now called the  $\Omega^-$ , belongs to Glashow and Sakurai.

Another crucial experimental development was the discovery of the  $\phi$  meson whose decay  $\phi \rightarrow K\bar{K}$  was SU(3) forbidden for a unitary singlet. The  $\phi$  had to be a mixture of an SU(3) singlet and octet produced by SU(3) breaking. There was also a peculiar suppression of the SU(3)-allowed  $\phi \rightarrow \rho\pi$  decay.

A simple dynamical model<sup>9)</sup> used the  $K - \pi$  mass difference to break SU(3), mix the SU(3) singlet and octet vector mesons  $\omega_1$  and  $\omega_8$  and forbid the  $\phi \rightarrow \rho\pi$

decay. The loop diagram connecting  $\omega_1$  and  $\omega_8$  vector mesons via vector-pseudoscalar intermediate states would cancel in the SU(3) symmetry limit

$$\omega_1 \rightarrow \rho\pi \rightarrow \omega_8; \quad \omega_1 \rightarrow K^*\bar{K} \rightarrow \omega_8 \quad \omega_1 \rightarrow \omega_8\eta_8 \rightarrow \omega_8 \quad (2.1)$$

However, the SU(3) breaking which lowers the pion mass far below the  $K$  and  $\eta$  enhances the  $\rho - \pi$  propagator in the transitions (2.1). The  $2 \times 2$  matrix (2.1) is thus dominated by transitions via the  $\rho - \pi$  intermediate state. Diagonalization of the loop-diagram matrix (2.1) with SU(3) breaking expressed by inserting experimental masses in the propagators produced mixed  $\omega_1 - \omega_8$  eigenstates with one eigenstate, the  $\phi$ , approximately decoupled from the  $\rho - \pi$  channel.

The U(3) description was rediscovered by Okubo who had noted that enlarging SU(3) symmetry to U(3) produced a meson nonet and that breaking the U(3) to  $U(2) \otimes U(1)$  gave what was later was called “ideal mixing”<sup>10)</sup> of the SU(3) singlet and octet vector mesons and naturally suppressed the  $\phi \rightarrow \rho\pi$  decay.

## 2.2. Quarks and Aces

These new experimental developments set the stage for a phenomenological investigation of the basic physics of the Goldberg-Ne'man model in which hadrons were constructed from a fundamental U(3) triplet with baryon number (1/3). The new phenomenology, called “quarks” by Gell-Mann<sup>11)</sup> and “aces” by Zweig<sup>12)</sup> showed remarkable agreement with the new experiments. The “Goldhaber Gap” in the experimental data showed the absence of the  $K^+N$  resonances and ruled out the 27-plet. The baryon spectrum confirmed the “tenfold way”. The vector nonet was found and the  $\phi - \omega$  mixing was observed and understood.

The statistics problem remained open. The  $\Delta^{++}$ ,  $\Delta^-$  and  $\Omega^-$  apparently violated fermi statistics by containing three identical spin-1/2 fermions in a spatially symmetric S-state coupled symmetrically to spin (3/2). This was solved by Greenberg<sup>13)</sup> by the introduction of parastatistics which later was seen to be equivalent to the introduction of another degree of freedom, now called color.

But the new phenomenology still had no sound theoretical basis. And there was no explanation for the peculiar hadron spectrum which had only quark-antiquark mesons and three-quark baryons and no “exotic states” with more quarks and antiquarks. Since both  $qq$  and  $q\bar{q}$  interactions were attractive, there seemed to be no simple way to prevent an antiquark from being bound to the three quarks in a baryon by the strong  $q\bar{q}$  attractive force to make a  $3q\bar{q}$  hadron.

## 2.3. The road to QCD

The new theory was supplied by Nambu,<sup>14)</sup> who showed that all strong interaction physics could be described by a non-abelian color-gauge theory with a Lagrangian now called the Lagrangian of QCD.

Nambu's QCD thus solved the three basic yet unsolved puzzles in strong interaction physics:<sup>15)</sup>

- 1) The triality puzzle. Why only states of triality zero appear as bound hadronic states.
- 2) The meson-baryon puzzle. Why the  $qq$  and  $q\bar{q}$  interactions are both attractive

but differ in strength in a way to bind the two-body  $q\bar{q}$  system and the three-body  $qqq$  system.

- 3) The exotics puzzle. Why only the  $q\bar{q}$  and  $qqq$  systems have bound states and all hadrons containing more quarks would decay into mesons and baryons

### §3. The crucial experiments and their phenomenological interpretations

#### 3.1. *The right experiment that killed the Sakata Model*

During the winter of 1961-62 The Weizmann Institute group was investigating experimental tests of unitary symmetry to distinguish between the Sakata and octet models.<sup>16)</sup> At a small conference on unitary symmetry organized by Abdus Salam at Imperial College, Harry Lipkin reported on the application to  $\bar{p}p$  annihilation into two mesons. Calculations on the blackboard with Salam showed that new experimental results from CERN strongly favored the Sakata model. Lipkin returned to Rehovot and discussions with Carl Levinson and Sydney Meshkov immediately revealed that the Salam - Lipkin calculation was based on incorrect values of SU(3) Clebsch-Gordan coefficients. The correct values gave a much more exciting and opposite result; a strong disagreement between the predictions of the Sakata model and the new experimental data.

This left a quandary. The paper had to be written immediately and sent for publication. Salam's name could not be omitted, since he had participated in the discussion that had led to this work. But including his name as an author would make him responsible for conclusions which were the exact opposite of his understanding from the meeting. Salam had already left for Pakistan and there were no postal relations between Israel and Pakistan. The manuscript was sent to Gerry Brown, then starting a new journal called "Physics Letters", with an explanation of the problem and carte blanche to use his own judgement. Brown contacted people at Imperial College and found that the error had independently been discovered by a member of Salam's group. The publication appeared in Volume 1 of Physics Letters with the three from Weizmann, Salam and his collaborator as joint authors.

#### 3.2. *The wrong experiment that led us to miss quarks*

Some time in the academic year 1961-62 Hayim Goldberg told about the work he had done with Yuval Ne'eman<sup>4)</sup> showing that the baryon octet could be constructed from three SU(3) triplets with baryon number (1/3). Whether or not you believe that these triplets are physical objects, this construction is interesting. The obvious (to us now; perhaps not then) is to note that three triplets could make the decuplet, but not the 27-plet. Placing the known resonances now called the  $\Delta(1238)$  and the  $\Sigma(1385)$  in the ten-dimensional representation of SU(3) and using the Gell-Mann-Okubo mass formula to calculate the masses would lead naturally to the prediction of the existence of the  $\Xi(1530)$  and the particle now called the  $\Omega^-$  with masses close to those eventually observed.

However this was not considered because the experimental data, now known to be wrong, indicated that the decay  $\Sigma(1385) \rightarrow \Sigma\pi$  was forbidden. This selection

rule forced the classification of the  $\Delta(1238)$  and the  $\Sigma(1385)$  in the 27-dimensional representation of SU(3) and not in the 10.

Sakurai<sup>6)</sup> had noted that the experimental value (1.1) implied a selection rule forbidding the  $\Sigma\pi$  decay. Incorporating a symmetry of hypercharge reflection called R-invariance into the “Eightfold Way” gave this selection rule<sup>6)</sup> and required putting the  $\Delta(1238)$  and the  $\Sigma(1385)$  in a 27-plet and not in a decuplet.

A detailed SU(3) description given by Okubo<sup>17)</sup> noted that the  $\Sigma(1385) \rightarrow \Sigma\pi$  decay was forbidden for a 27-plet  $\Sigma(1385)$  and the result for the decuplet strongly disagreed with experiment.

$$\frac{BR[\Sigma(27) \rightarrow \Sigma\pi]}{BR[\Sigma(27) \rightarrow \Lambda\pi]} = 0; \quad \frac{BR[\Sigma(10) \rightarrow \Sigma\pi]}{BR[\Sigma(10) \rightarrow \Lambda\pi]} = 15\% \quad (3.1)$$

The Weizmann group<sup>16)</sup> saw that the 27-plet classification was needed to fit experiment and immediately called for experimental searches for the positive strangeness resonances expected in the 27-plet but not in the decuplet.

Unfortunately these data were wrong, there was no selection rule and no  $K^+N$  resonance. The new data much later confirmed the decuplet branching ratio.

$$BR[\Sigma(1385) \rightarrow \Sigma\pi] = (11.7 \pm 1.5)\%; \quad BR[\Sigma(1385) \rightarrow \Lambda\pi] = (87.0 \pm 1.5)\% \quad (3.2)$$

$$\frac{BR[\Sigma(1385) \rightarrow \Sigma\pi]}{BR[\Sigma(1385) \rightarrow \Lambda\pi]} = (13 \pm 2)\% \quad (3.3)$$

### 3.3. Further “right experiments” that confirmed the triplet model

New “right” experiments found the  $\Xi^*$  (1530) and revealed the complete absence of the positive strangeness KN resonances expected in the 27-plet, called the Goldhaber Gap. But their implications for the decuplet classification were not noted until the  $\Xi^*$  was found, rather than noting that the existence and mass of the  $\Xi^*$  should have been predicted. The wrong value (1.1) for the  $\Sigma\pi$  decay prevented seeing the obvious implications of the Goldberg-Ne'eman breakthrough.

That the  $\Xi^*$  mass fit exactly the prediction of the Gell-Mann-Okubo mass formula for a decuplet was immediately noted by Glashow and Sakurai.<sup>8)</sup> Their “tenfold way” paper was immediately noted and used<sup>18)</sup> to make SU(3) predictions for decuplet production in meson-baryon reactions, and the possibility of making  $Z^- \bar{Z}^+$  pairs in nucleon-antinucleon annihilation. This first published prediction<sup>8)</sup> for the existence of this particle was then already acknowledged as  $Z^-$  in published literature, but is now generally overlooked. I once asked J. J. Sakurai why they never claimed credit for the first publication of this prediction. His response was that they were highly embarrassed by their paper because they had blindly substituted into the Gell-Mann-Okubo mass formula without noting that this becomes an equal-spacing rule for a decuplet.

The finding of this particle, now called the  $\Omega^-$ , together with the “Goldhaber Gap” confirmed the “Tenfold Way” now called the decuplet classification and led to the general acceptance of the Goldberg-Ne'eman triplet model, now called the quark model. But the wrong experimental value (1.1) for the  $\Sigma\pi$  decay remained an

obstacle to this interpretation until better experiments showed agreement with the decuplet prediction.

#### §4. The impact of the Sakata Model beyond the original $pn\Lambda$ hadron Model

##### 4.1. The “sakaton” - teaching group theory to particle physicists

The  $SU(3)$  “unitary symmetry” group used in the Sakata model was also used in the Gell-Mann-Ne’eman octet model called the “Eightfold Way”. But Murray Gell-Mann, like most particle theorists in the U.S. and Europe, knew no group theory at the time. Group theory was viewed as irrelevant mathematics (Die Gruppenpest) which had no use in particle physics. And nuclear and condensed matter physics were disregarded as “dirt physics” and “squalid state physics”.<sup>20)</sup> The particle theorists saw isospin as rotations in an abstract three-dimensional space and spent eight years searching for a higher symmetry in rotations in spaces of higher dimensions.<sup>19)</sup> One might say that they called  $SU(3)$  symmetry the eightfold way because it took them eight years to learn that isospin and strangeness are  $SU(2) \otimes U(1)$ . They did not know that isospin is also  $SU(2)$  and that  $SU(3)$  is a natural symmetry to include  $SU(2)$  and  $U(1)$ .

The existence and algebra of unitary groups and in particular  $SU(3)$ , although unknown to Gell-Mann and his American and European colleagues in particle physics, was well known in the nuclear physics community. However  $SU(3)$  was used in nuclear physics as the invariance group of the three-dimensional harmonic oscillator, and its representations and Clebsch-Gordan coefficients were always classified using the subgroup  $O(3)$  of rotations in three dimensions. Levinson, Lipkin and Meshkov knew this classification very well, but relied on the Sakata model papers<sup>2),3)</sup> to obtain the  $SU(3)$  Clebsch-Gordan coefficients using the subgroup  $SU(2) \times U(1)$ . They used the word “sakaton” as a general name for a fundamental triplet of  $SU(3)$ .

The book “Lie Groups for Pedestrians”<sup>21)</sup> arose from the need for a simple set of lectures to teach the necessary Lie algebras to particle and nuclear physicists. To present  $SU(3)$  in a simple way, the Sakata model was used with the name “sakaton” for the  $pn\Lambda$  triplet.

##### 4.2. Renaissance of the Sakata model in hypernuclear physics

In 1971 a physical motivation was presented for using a dynamical symmetry of the Sakata model type in hypernuclear physics was given.<sup>22)</sup> The  $pn\Lambda$  triplet was considered as the constituents of hypernuclei with  $SU(3)$  symmetry. Earlier works on  $SU(3)$  symmetry applications to hypernuclei had used the octet  $SU(3)$  version in which the  $\Lambda$  and  $\Sigma$  are degenerate. But the 80 MeV  $\Lambda - \Sigma$  splitting was too big for nuclear and hypernuclear excitations.

A “strangeness analog state” obtained by a U-spin operation on all neutrons in the nuclear ground state was defined. The suggestion<sup>23)</sup> that this strangeness analog state had been observed in the first  $(K^-, \pi^-)$  experiments done in the CERN PS. turned out to be wrong. Dalitz and Gal<sup>24)</sup> narrowed the symmetry to only the

valence neutrons. The Sakata SU(3) symmetry was combined with Pauli spin SU(2) to SU(6) supermultiplets which include both nuclei and hypernuclei, but only for a particular shell correspondence. This was a natural extension of Wigner's SU(4) supermultiplet theory for ordinary (light) nuclei. In particular the  ${}^9_{\Lambda}\text{Be}$  hypernucleus was analyzed.<sup>25)</sup> A very interesting consequence of this work concerned a particularly symmetric state in the excited hypernuclear spectrum termed "supersymmetric". This state has been discovered in  ${}^9_{\Lambda}\text{Be}$ .

This supersymmetric state concept was rediscovered in 1983 and termed a "genuinely hypernuclear state".<sup>26)</sup> A review<sup>27)</sup> cites both the Dalitz-Gal theoretical work<sup>24)</sup> in 1976 and the Japanese work<sup>26)</sup> in the 1980s.

### §5. Details of the Sakata selection rule forbidding $p\bar{p} \rightarrow K_L K_S$

In the Sakata model annihilation into charged kaon pairs and charged pion pairs is allowed but annihilation at rest into neutral kaon pairs is forbidden. This prediction was in strong disagreement with experiment, which showed that  $K_L - K_S$  pairs were produced at comparable rates with charged kaon and pion pairs. There is a very simple "pedestrian" explanation of this selection rule.

In the Sakata model the neutral kaons are made of neutrons and  $\Lambda$ 's and their antiparticles and contain no protons nor antiprotons. The charged pions and kaons all contain a proton or antiproton. Thus a proton-antiproton system can become two charged pions or kaons by creating a single additional neutron-antineutron or  $\Lambda$ -anti- $\Lambda$  pair which combines with the initial proton and antiproton to form the two final mesons. This cannot occur for the neutral kaon pair final state.

The selection rule can also be seen as an SU(3) rotation of the isospin and parity selection rule forbidding the annihilation of odd-parity  $\Lambda\bar{\Lambda}$  states into two pions. The  $\Lambda\bar{\Lambda}$  state has isospin zero and the isospin zero states of two pions are symmetric under interchange of the two pions and have even parity.

$$\bar{\Lambda}\Lambda \rightarrow \pi^+ + \pi^- \text{ (forbidden for odd parity)} \quad (5.1)$$

In the Sakata model there is an SU(3) symmetry transformation which interchanges protons and  $\Lambda$ 's everywhere. Under this transformation the charged pions,  $(p\bar{n})$  and  $(\bar{p}n)$  become neutral kaons  $(\Lambda\bar{n})$  and  $(\bar{\Lambda}n)$  and the selection rule becomes

$$\bar{p}p \rightarrow K^o + \bar{K}^o \text{ (forbidden for odd parity)} \quad (5.2)$$

Although this selection rule holds only for odd parity states, the annihilation into  $K_L K_S$  pairs is forbidden for all partial waves, since even parity  $K\bar{K}$  pairs are allowed only to decay only into the  $K_L K_L$  and  $K_S K_S$  decay modes, but never into  $K_L K_S$ . In the octet model and the quark model there is no such selection rule, as the analogous transformation on pions and kaons via interchanging u and s quarks mixes  $\Lambda$ 's and  $\Sigma$ 's rather than  $\Lambda$ 's and protons.

Salam's collaborator, Munir Ahmed Rashid, appeared on this paper<sup>5)</sup> as R. A. Munir instead of M. A. Rashid. This confusion arises because Pakistani Moslem names are often words joined together in a phrase with a well-defined meaning,

rather than a Christian name and a family name. Abdus Salam, for example, can mean a servant of peace. We conclude with the hope that these days we should all try to be servants of peace.

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